Advanced Math **Section 3.3 Notes** **Name:**
Rational Root Theorem

Learning Target: ☺ *Finding the zeros of a polynomial using the Rational Root Theorem.*

Warm Up: Use synthetic division to see if c is a zero of $P(x)$.

a. $P(x) = 4x^3 - 10x^2 - 8x + 6, \quad c = 1$ **b.** $P(x) = x^4 - 2x^2 - 100x - 75, \quad c = 5$

A polynomial function P of degree n has a most _____ zeros, including _____.

Rational Zero Theorem: We will do an **actual example** to show this theorem!

Given $P(x) = x^3 + 2x^2 - x - 2$

$p =$ _____ (_____) $q =$ _____ (_____)

Factors of p: Factors of q:

- Possible rational zeros is the following list:
  - All possible_____ _____ **divided by**

    all possible _____
  - Complete the list by putting a _____ sign in front of each possible rational zero

List of possible rational zeros from the example above:

Steps to Finding Real Zeros of $P(x)$.

1. Do _____ using the list you just generated to find the rational zeros.

2. Once you get the quotient down to a _____, _____ to solve for the remaining rational zeros.

3. If the quadratic cannot be factored, the remaining zeros are either _____ or _____ and should be found using the _____ _____.

4. Once you find a rational zero, if what's left is still greater than a quadratic, you should always check for _____.

5. The _____ equals the _____ of zeros!!

Turn the page over to continue the example for the Rational Root Theorem

Example 1: Given $P(x) = x^3 + 2x^2 - x - 2$ Degree = _____ Number of Zeros = _____

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$p = \mathbf{2}$ $q = \mathbf{1}$

Factors of p: $\mathbf{1}, \mathbf{2}$ Factors of q: $\mathbf{1}$

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros: $\pm \mathbf{1}, \pm \mathbf{2}$

Start the synthetic division with $+\mathbf{1}$ What are the rational zeros of $P(x)$?_____

Example 2: Given $P(x) = x^3 + 2x^2 - 5x - 6$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

Example 3: Given $P(x) = x^4 + 4x^3 - x^2 - 16x - 12$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

Example 4: Given $P(x) = 2x^3 + 9x^2 + 10x + 3$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

Key Ideas:

Advanced Math **Section 3.3 HW** **Name:**
Rational Root Theorem

1. Given $P(x) = x^3 + 3x^2 - 6x - 8$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

2. Given $P(x) = x^3 - 3x - 2$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

Don't forget to put a zero where x^2 belongs!

3. Given $P(x) = 2x^3 + 9x^2 - 2x - 9$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

4. Given $P(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

5. Given $P(x) = x^3 - 19x - 30$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

Don't forget to put a zero where x^2 belongs!

6. Given $P(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$ Degree = _____ Number of Zeros = _____

$p =$ _____ $q =$ _____

Factors of p: Factors of q:

Possible rational zeros is the list of all possible factors of p divided by all possible factors of q. Put a $\pm$ sign in front of each possible rational zero!

List of possible rational zeros:

Start the synthetic division with $+1$ What are the rational zeros of $P(x)$?_____

Don't forget to check for multiplicities!

Ex 1: $1, \ -1, \ -2$ **Ex 2:** $-1, \ -3, \ 2$ **Ex 3:** $-1, \ 2, \ -2, \ -3$ **Ex 4:** $-1, \ -3, \ -\frac{1}{2}$

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1. $-1, \ 2, \ -4$ **2.** -1 multiplicity $2, \ 2$ **3.** $1, \ -1, \ -\frac{9}{2}$

4. $1, \ -1, \ -2, \ \frac{1}{2}$ **5.** $-2, \ -3, \ 5$ **6.** -1 multiplicity $2, \ 2, \ -2$