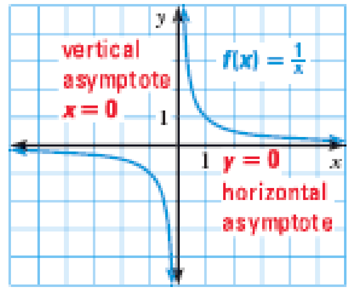
Algebra 2 – Graphing Rational Functions – Chapter 8, Sections 2 and 3

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called *branches*. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \frac{a}{x}$ $(a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.



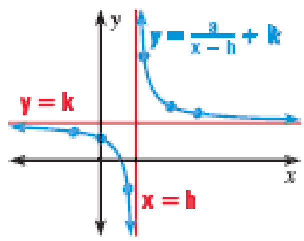
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \frac{a}{x-h} + k$, follow these steps:

STEP 1 **Draw** the asymptotes $x = h$ and $y = k$.

STEP 2 **Plot** points to the left and to the right of the vertical asymptote.

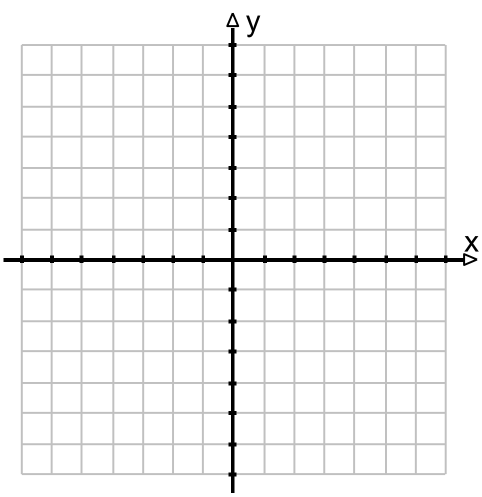
STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

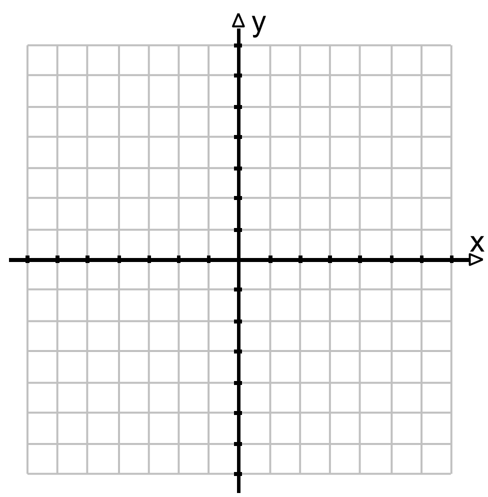


Some examples: *Graph, list asymptotes, and describe domain and range*

$$y = \frac{3}{x+1} - 2$$

$$f(x) = \frac{-2}{x-6} + 4$$





Now graph these on your calculator: *Are they the same kind of shape?*

$$y = \frac{x}{x-2}$$

$$y = \frac{2x-5}{x+4}$$

$$f(x) = \frac{2x}{x^2-1}$$

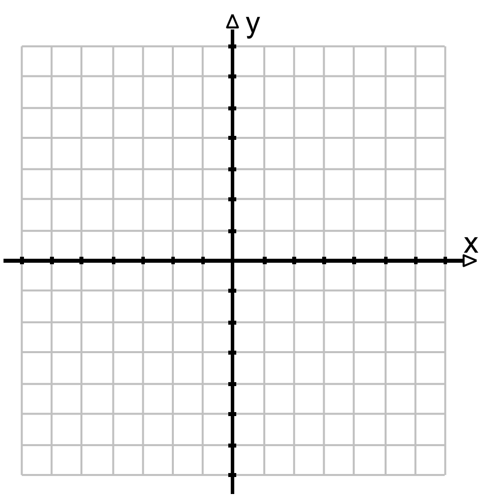
Steps for graphing a not-so-simple rational function:

1. Make sure the numerator and denominator are each in standard form.
2. Factor both the numerator and denominator. Cancel any common factors.
3. Plug in 0 as the x value to find the y-intercept.
4. Set the numerator equal to zero and solve. The solutions are the x-intercepts.
5. Set the denominator equal to zero and solve. The solutions are the vertical asymptotes.
6. The graph will have at most one horizontal asymptote. To find it, compare the degrees of the numerator and denominator:
   a) If the numerator's degree is smaller than the denominator's degree, then $y = 0$ is the horizontal asymptote.
   b) If the numerator's degree is larger, there is no horizontal asymptote.
   c) If the degrees are the same, then the horizontal asymptote has the equation $y = a / b$, where a and b are the leading coefficients of the numerator and denominator.
7. Did you cancel any common factors in step 2? If so, set those factors equal to zero and solve. Those x values are places where the graph has a hole. You can find the y value that goes with each of those x values to see the specific locations of the holes.
8. Be able to graph, list the asymptotes and holes, describe the domain, and explain the end behavior of the graph. (The range is often more complicated, so it's OK to leave it out.)

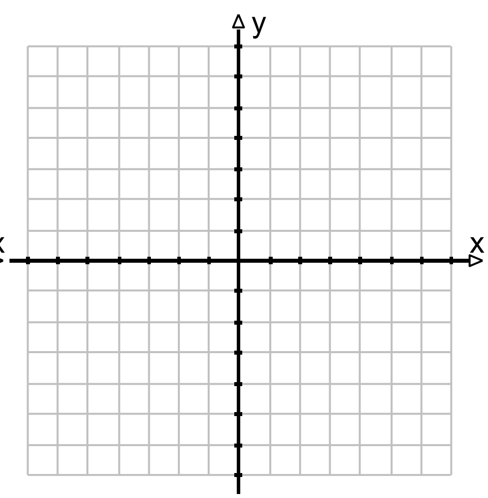
More examples:

Graph, list asymptotes and holes, and describe domain and end behavior

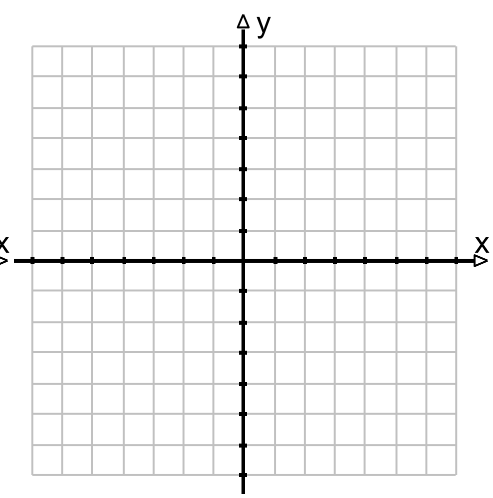
$y = \dfrac{x^2+1}{-3x+6}$



$f(x) = \dfrac{x^2+x-12}{x^2-4}$



$y = \dfrac{x^2-9}{5x-15}$



Graph these on your calculator:

Make sure you can list all relevant facts

$f(x) = \dfrac{3}{x^2-x-6}$ $\quad y = \dfrac{2x}{x^2-5x}$ $\quad y = \dfrac{x}{x^2+4}$ $\quad f(x) = \dfrac{x^2}{x^2+4x+3}$