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Lesson 5: The Tortoise and the Hare

Solidify Understanding

Jump Start

Which function has the greatest average rate of change in the interval given?

A. $f(x) = 2x - 3$

Interval $[-2, 3]$

B. $g(x) = x(x - 3)$

Interval $[2, 4]$

C. $h(x) = 3^x$

Interval $[0, 2]$

Learning Focus

Compare quadratic and exponential functions.

Determine which type of function, quadratic or exponential, grows faster in a given interval.

How does the growth of a quadratic function compare to the growth of an exponential function?

How can tables, graphs, and equations help to understand how quadratic and exponential functions grow?

Open Up the Math

Launch, Explore, Discuss

In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function: $d = t^2$ (d in meters and t in seconds)

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise, including the head start, is given by the function: $d = 2^t$ (d in meters and t in seconds)

1. At what time does the hare catch up to the tortoise?

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2. If the race course is very long, who wins: the tortoise or the hare? Why?

3. At what time(s) are they tied?

4. If the race course were 15 meters long who wins: the tortoise or the hare? Why?

5. Use the properties $d = 2^t$ and $d = t^2$ to explain the speeds of the tortoise and the hare in the following time intervals:

Interval	**Tortoise** $d = 2^t$	**Hare** $d = t^2$
$[0, 2)$		
$[2, 4)$		
$[4, \infty)$		

Ready for More?

In the case of the tortoise and the hare, the exponential function exceeded the quadratic function for large values of t. Would this be the case if the quadratic function were doubled? Compare $y = 2x^2$ and $y = 2^x$ and decide which function grows faster for large values of x. Explain why your answer is correct.

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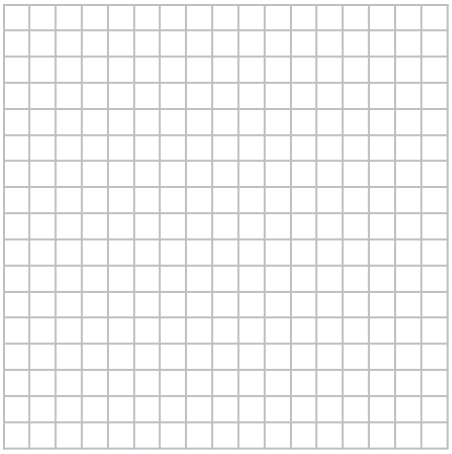
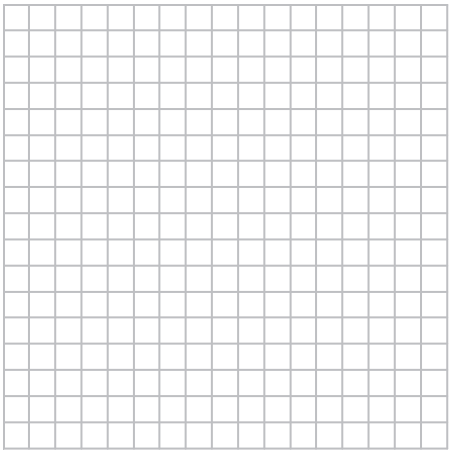
Takeaways

	Quadratic Function	Exponential Function
Shape of Graph:		
Equation:		
Table:		
Rate of Change:		

Lesson Summary

In this lesson we compared quadratic and exponential functions. We learned that in some intervals for small values of x, quadratic functions may be greater than exponential functions. For large values of x, exponential functions greatly exceed quadratic functions because of the difference in their rates of change.

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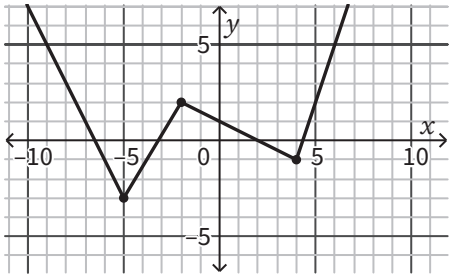
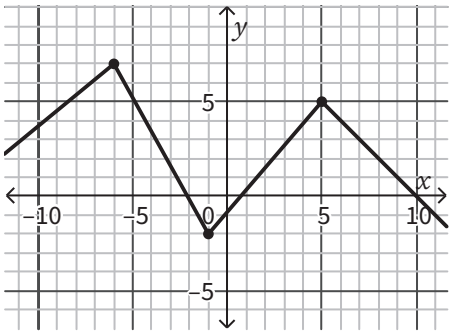
Retrieval

Determine whether each representation is a function or not a function. If it is not a function describe what you would do to change it to make it a function.

1. $\{(2,7), (3,9), (4,8), (5,5), (6,-3), (7,5), (3,7)\}$

2. 

State the domain and range of each graph. Use interval notation where appropriate.

3. 

4.