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Lesson 2: Flipping Ferraris

Solidify Understanding

Learning Focus

Understand the inverse of a quadratic function.

Determine the relationship between the domain and range of a function and its inverse.

Understand when the inverse of a function is also a function.

Do all functions have inverses? If so, are their inverses always functions?

What type of relation is the inverse of a quadratic function?

Open Up the Math

Launch, Explore, Discuss

When people first learn to drive, they are often told that the faster they are driving, the longer it will take to stop. So, when you're driving on the freeway, you should leave more space between your car and the car in front of you than when you are driving slowly through a neighborhood. Have you ever wondered about the relationship between how fast you are driving and how far you travel before you stop, after hitting the brakes?

1. Think about it for a minute. What factors do you think might make a difference in how far a car travels after hitting the brakes?

There has actually been quite a bit of experimental work done (mostly by police departments and insurance companies) to be able to mathematically model the relationship between the speed of a car and the braking distance (how far the car goes until it stops after the driver hits the brakes).

2. Imagine your dream car. Maybe it is a Ferrari 550 Maranello, a super-fast Italian car. Experiments have shown that on smooth, dry roads, the relationship between the braking distance (d) and speed (s) is given by $d(s) = 0.03s^2$. Speed is given in miles/hour and the distance is in feet.

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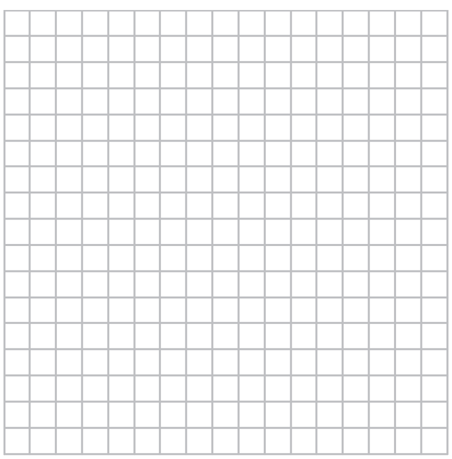
a. How many feet should you leave between you and the car in front of you if you are driving the Ferrari 55 mph?

b. What distance should you keep between you and the car in front of you if you are driving 100 mph?

c. If an average car is about 16 feet long, about how many car lengths should you have between you and the car in front of you if you are driving 100 mph?

d. It makes sense to a lot of people that if the car is moving at some speed and then goes twice as fast, the braking distance will be twice as far. Is that true? Explain why or why not.

3. Graph the relationship between braking distance $d(s)$ and speed (s).



4. According to the Ferrari Company, the maximum speed of the car is about 217 mph. Use this information to describe all the mathematical features of the relationship between braking distance and speed for the Ferrari modeled by $d(s) = 0.03s^2$. Your answer should include the domain and range, any intercepts, maximums or minimums, and the intervals of increase and decrease.

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5. What if the driver of the Ferrari 550 was cruising along and suddenly hit the brakes to stop because she saw a cat in the road? She skidded to a stop and, fortunately, missed the cat. When she got out of the car she measured the skid marks left by the car so she knew that her braking distance was 31 ft.

   a. How fast was she going when she hit the brakes?

   b. If she didn't see the cat until she was 15 feet away, what is the fastest speed she could be traveling before she hit the brakes if she wants to avoid hitting the cat?

6. Part of the job of police officers is to investigate traffic accidents to determine what caused the accident and which driver was at fault. They measure the braking distance using skid marks and calculate speeds using the mathematical relationships, just like we have here, although they often use different formulas to account for various factors, such as road conditions. Let's go back to the Ferrari on a smooth, dry road since we know the relationship. Create a table that shows the speed the car was traveling based upon the braking distance.

7. Write an equation of the function $s(d)$, that gives the speed the car was traveling for a given braking distance.

8. Graph the function $s(d)$, and describe its features, including domain, range, any intercepts, maximums, minimums, or intervals of increase or decrease.

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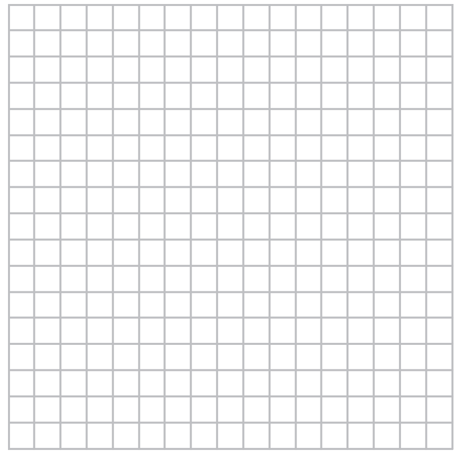
Features of $s(d)$ include:



9. What do you notice about the graph of $s(d)$ compared to the graph of $d(s)$? What is the relationship between the functions $d(s)$ and $s(d)$?

10. Consider the function $d(s) = 0.03s^2$ over the domain of all real numbers, not just the domain of this problem situation. How does the graph change from the graph of $d(s)$ in problem #3?

11. How does changing the domain of $d(s)$ change the graph of the inverse of $d(s)$?

12. Is the inverse of $d(s)$ a function? Justify your answer.

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Ready for More?

Find a domain that makes the function invertible, and then find the inverse.

$$f(x) = \frac{1}{2}(x+3)^2 - 4$$

Takeaways

Characteristics of functions and their inverses:

Adding Notation, Vocabulary, and Conventions

Invertible function:

Vocabulary

- domain
- interval of increase or decrease
- **invertible function**
- maximum / minimum
- **one-to-one function**
- quadratic equations
- quadratic function
- range of a function

Bold terms are new in this lesson.

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Lesson Summary

In this lesson, we examined a quadratic function and its inverse. We found characteristics of inverse functions that are common across function types. We learned that some functions are invertible and that if a function is not invertible, the domain can be restricted to make it invertible.

Retrieval

Solve for x.

1. $\sqrt{x^2 + x - 11} = 3$

2. Given: $f(x) = 5x$ and $g(x) = x^2 + 2x$

   Find $f(g(x))$