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Lesson 3: Tracking the Tortoise

Solidify Understanding

Learning Focus

Represent the inverse of an exponential function.

Compare the inverse relationship for an exponential function with the inverses of linear and quadratic functions.

Determine if a function is invertible.

What properties of inverse functions for linear and quadratic functions are the same for exponential functions? Can these properties be generalized to any function and its inverse?

Open Up the Math

Launch, Explore, Discuss

You may remember a previous lesson about the famous race between the tortoise and the hare. In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race.

Today we will consider the journey of the tortoise in the race. Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. Won't the hare be surprised when he finds that Shellie is a wonder-tortoise with supersonic exponential speed? The distance from the starting line of the tortoise, including the head start, is given by the function:

$d(t) = 2^t$ (d in meters and t in seconds)

The tortoise family decides to watch the race from the sidelines so they can see their darling tortoise sister, Shellie, prove the value of persistence.

1. How far away from the starting line must the family be so they are located in the right place for Shellie to run by 5 seconds after the beginning of the race? After 10 seconds?

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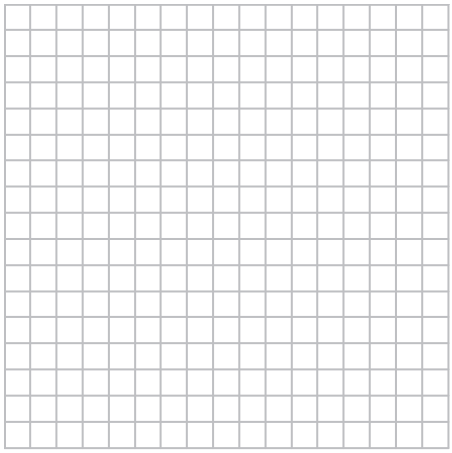
2. Describe the graph of $d(t)$, Shellie's distance at time t. What are the important features of $d(t)$?

3. If the tortoise family plans to watch the race at 64 meters away from Shellie's starting point, how long will they have to wait to see Shellie run past?

4. How long must they wait to see Shellie run by if they stand $1{,}024$ meters away from her starting point?

5. Draw a graph that shows how long the tortoise family will wait to see Shellie run by at a given location from her starting point.



6. How long must the family wait to see Shellie run by if they stand 220 meters away from her starting point?

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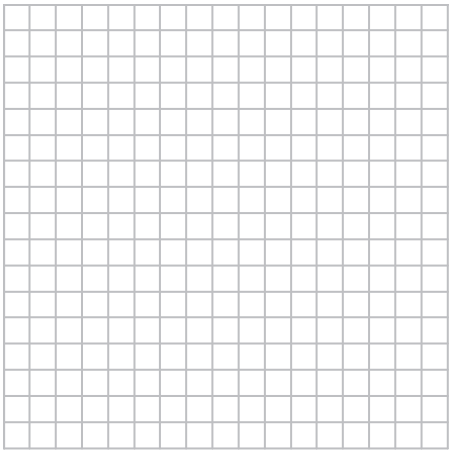
7. What is the relationship between $d(t)$ and the graph that you have just drawn? How did you use $d(t)$ to draw the graph in #5?

8. Consider the function $f(x) = 2^x$.

a. What are the domain and range of $f(x)$? Is $f(x)$ invertible?

b. Graph $f(x)$ and $f^{-1}(x)$ on the grid.



c. What are the domain and range of $f^{-1}(x)$?

9. If $f(3) = 8$, what is $f^{-1}(8)$? How do you know?

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10. If $f(\frac{1}{2}) = 1.414$, what is $f^{-1}(1.414)$? How do you know?

11. If $f(a) = b$, what is $f^{-1}(b)$? Will your answer change if $f(x)$ is a different function? Explain.

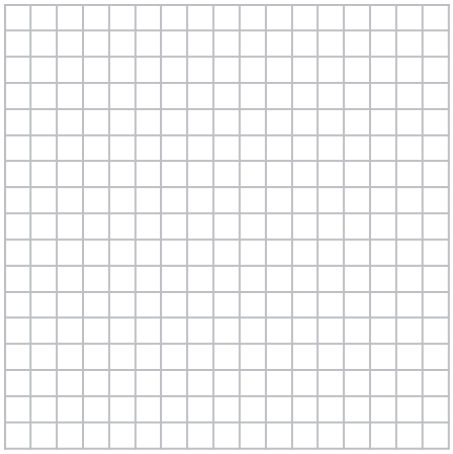
Ready for More?

Find the graph of the inverse of $f(x) = 10^x$.

What features does this graph have in common with the graph of the inverse of $f(x) = 2^x$?

Takeaways

Exponential functions and their inverses:



Inverse functions undo each other:

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Vocabulary

- **logarithmic function (logarithm)**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we modeled the inverse of an exponential function to determine its features. We learned that this type of function is called a logarithmic function, which we will learn more about in Unit 2. We also discussed a way to describe the input-output relationship of inverse functions using mathematical notation.

Retrieval

1. Solve for the value of x.

   $3^{4x+5} = 3^{7x-13}$

2. Calculate $f(a+b)$, given that $f(x) = x^2 + 4x - 3$.