NAME DATE PERIOD

Lesson 1: Log Logic

Develop Understanding

Learning Focus

Find exact and estimated values of logarithms.

Make conjectures about logarithm properties.

What is a logarithm?

How can we tell if one log expression is bigger than another?

How can we estimate log values?

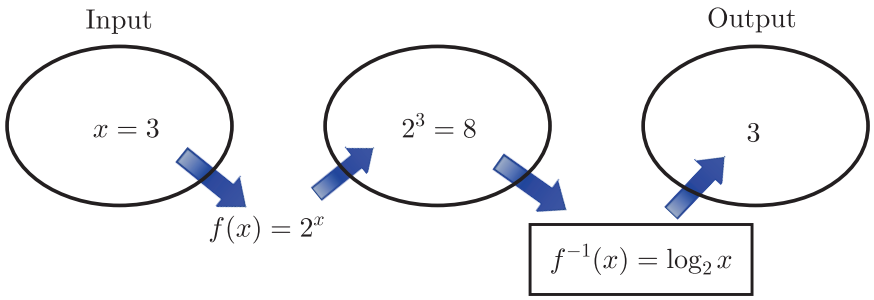
How do logs relate to exponents?

Open Up the Math

Launch, Explore, Discuss

We began thinking about logarithms as inverse functions for exponentials in *Tracking the Tortoise*. Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.

We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one shown:



We could summarize this relationship by saying:

$2^3 = 8$ so, $\log_2 8 = 3$

Logarithms can be defined for any base associated with an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

NAME DATE PERIOD

$10^1 = 10$ so, $\log_{10} 10 = 1$

$10^2 = 100$ so, $\log_{10} 100 = 2$

$10^3 = 1000$ so, $\log_{10} 1000 = 3$

The notation may seem different, but you can see the inverse pattern where the inputs and outputs switch.

The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns related to logarithms.

1. Place the following expressions on the number line. Explain how you knew where to place each expression.

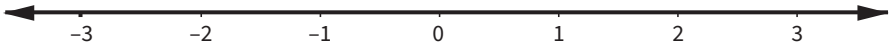
a. $\log_3 3$

b. $\log_3 9$

c. $\log_3 \frac{1}{3}$

d. $\log_3 1$

e. $\log_3 \frac{1}{9}$



Explain:

A conjecture about logarithmic expressions based on problem 1:

2. a. $\log_3 81$

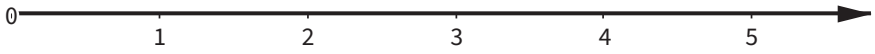
b. $\log_{10} 100$

c. $\log_8 8$

d. $\log_5 25$

e. $\log_2 32$

NAME DATE PERIOD

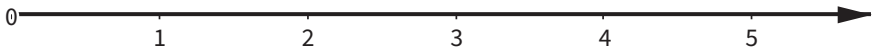


Explain:

A conjecture about logarithmic expressions based on problem 2:

Pause and Reflect

3. a. $\log_7 7$
b. $\log_9 9$
c. $\log_{11} 1$
d. $\log_{10} 1$



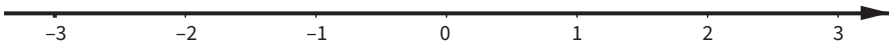
Explain:

A conjecture about logarithmic expressions based on problem 3:

4. a. $\log_2 \left(\frac{1}{4}\right)$
b. $\log_{10} \left(\frac{1}{1000}\right)$
c. $\log_5 \left(\frac{1}{125}\right)$

NAME DATE PERIOD

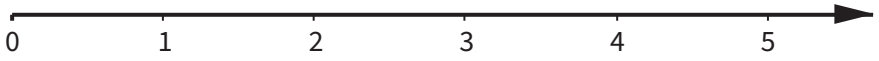
d. $\log_6\left(\frac{1}{6}\right)$



Explain:

A conjecture about logarithmic expressions based on problem 4:

5. a. $\log_4 16$
b. $\log_2 16$
c. $\log_8 16$
d. $\log_{16} 16$

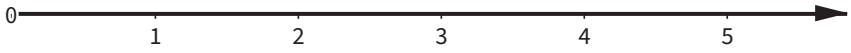


Explain:

A conjecture about logarithmic expressions based on problem 5:

6. a. $\log_2 5$
b. $\log_5 10$
c. $\log_6 1$
d. $\log_5 5$
e. $\log_{10} 5$

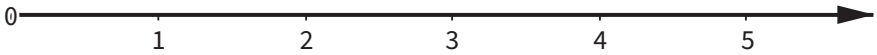
NAME DATE PERIOD



Explain:

A conjecture about logarithmic expressions based on problem 6:

7. a. $\log_{10} 50$
b. $\log_{10} 150$
c. $\log_{10} 1000$
d. $\log_{10} 500$

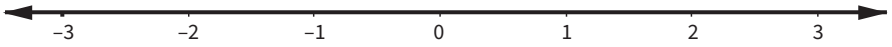


Explain:

A conjecture about logarithmic expressions based on problem 7:

8. a. $\log_3 3^2$
b. $\log_5 5^{-2}$
c. $\log_6 6^0$
d. $\log_4 4^{-1}$
e. $\log_2 2^3$

NAME DATE PERIOD



Explain:

A conjecture about logarithmic expressions based on problem 8:

Based on your work with logarithmic expressions, determine whether each of these statements is always true, sometimes true, or never true. If the statement is sometimes true, describe the conditions that make it true. Explain your answers.

9. The value of $\log_b x$ is positive.

Explain:

10. $\log_b x$ is not a valid expression if x is a negative number.

Explain:

11. $\log_b 1 = 0$ for any base, $b > 0$.

Explain:

NAME DATE PERIOD

12. $\log_b b = 1$ for any $b > 0$.

Explain:

13. $\log_2 x < \log_3 x$ for any value of x.

Explain:

14. $\log_b b^n = n$ for any $b > 0$.

Explain:

Ready for More?

Find the value of: $b^{\log_b n}$.

For what values of b and n is your answer correct?

NAME DATE PERIOD

Takeaways

Properties of Logarithms:

Vocabulary

- **argument of a logarithm**
- **base of a logarithm**

Bold terms are new in this lesson.

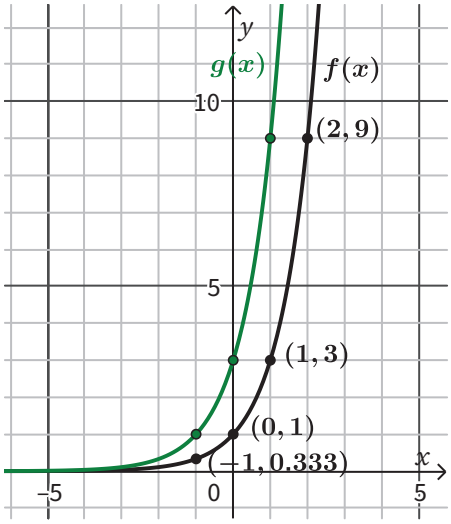
Lesson Summary

In this lesson, we found exact and approximate values for logarithmic expressions. We drew conclusions about properties of logarithms that are always true. We also identified characteristics of logarithmic functions that will help us to graph logarithms in the next lesson.

NAME DATE PERIOD

Retrieval

1. The graph of $f(x) = 3^x$ is shown. Key points on the graph of $f(x)$ have been labeled. The green graph, $g(x)$, shows a new graph that is a transformation of $f(x)$.



a. Three points are noted on $g(x)$. What are the coordinates of each point?

b. Describe in words the transformation on $f(x)$ that has generated $g(x)$.

c. Write the equation for $g(x)$.

2. Rewrite $\frac{24^{-1}}{16^{-1}}$ as an integer or a fraction in lowest terms.