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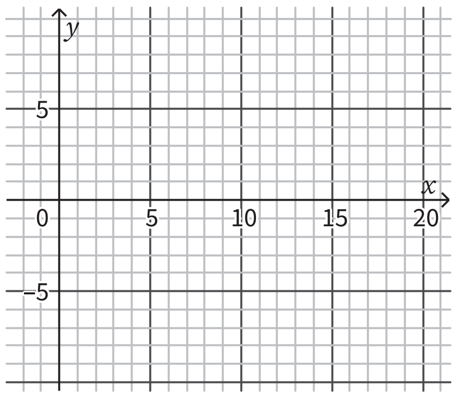
Lesson 3: Chopping Logs

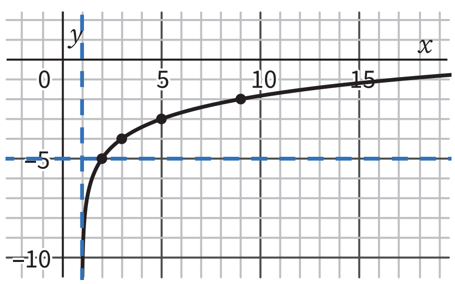
Solidify Understanding

Jump Start

1. Graph the equation: $y = 2 + \log_2(x + 3)$



2. Write the equation for the base 2 logarithmic function with the given graph:



Learning Focus

Use graphs to discover properties of logarithms.

Justify conjectures about logarithms.

Can the graphs of logarithmic functions give us insight into some of the algebraic properties of logarithms?

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Open Up the Math

Launch, Explore, Discuss

Abe and Mary are working on their math homework together when Abe has a brilliant idea.

Abe: I was just looking at this log function that we graphed in *Falling Off A Log*:

$y = \log_2(x + b)$.

I started to think that maybe I could just "distribute" the log so that I get:

$y = \log_2 x + \log_2 b$.

I guess I'm saying that I think these are equivalent expressions, so I could write it this way:

$\log_2(x + b) = \log_2 x + \log_2 b$.

Mary: I don't know about that. Logs are tricky and I don't think that you're really doing the same thing here as when you distribute a number.

1. What do you think? How can you verify if Abe's idea works?

2. If Abe's idea works, give some examples that illustrate why it works. If Abe's idea doesn't work, give a counterexample.

Abe: I just know there is something going on with these logs. I just graphed $f(x) = \log_2(4x)$. Here it is:

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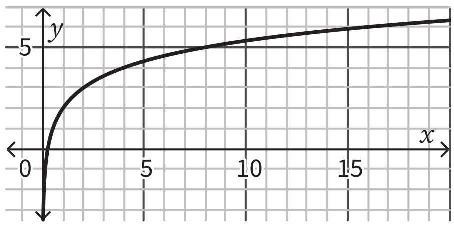

It's weird because I think this graph is just a translation of $y = \log_2 x$. Is it possible the equation of this graph could be written more than one way?

3. How would you answer Abe's question? Are there conditions that could allow the same graph to have different equations?

Mary: When you say, "a translation of $y = \log_2 x$" do you mean that it is just a vertical or horizontal shift? What could that equation be?

4. Find an equation for $f(x)$ that shows it to be a horizontal or vertical shift of $y = \log_2 x$.

Mary: I wonder why the vertical shift turned out to be up 2 when the x was multiplied by 4. I wonder if it has something to do with the power that the base is raised to, since this is a log function. Let's try to see what happens with $y = \log_2(8x)$ and $y = \log_2(16x)$.

5. Try to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a. $y = \log_2(8x)$

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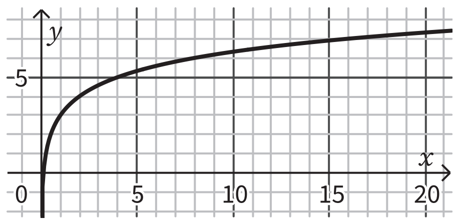
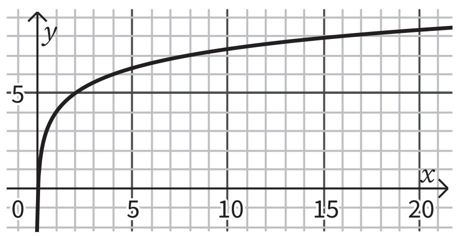


Equivalent equation:

b. $y = \log_2 (16x)$



Equivalent equation:

Mary: Oh my gosh! I think I know what is happening here! Here's what we see from the graphs:

$$\log_2(4x) = 2 + \log_2 x$$
$$\log_2(8x) = 3 + \log_2 x$$
$$\log_2(16x) = 4 + \log_2 x$$

Here's the brilliant part: We know that $\log_2 4 = 2$, $\log_2 8 = 3$, and $\log_2 16 = 4$. So:

$$\log_2(4x) = \log_2 4 + \log_2 x$$
$$\log_2(8x) = \log_2 8 + \log_2 x$$
$$\log_2(16x) = \log_2 16 + \log_2 x$$

I think it looks like the "distributive" thing that you were trying to do, but since you can't really distribute a function, it's really just a log multiplication rule. I guess my rule would be:

$$\log_2 (ab) = \log_2 a + \log_2 b$$

6. How can you express Mary's rule in words?

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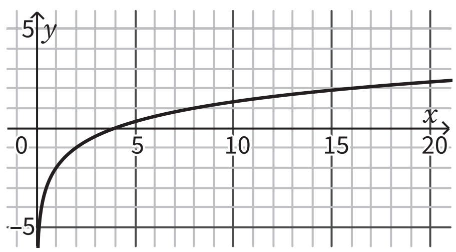
7. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counterexample.

Mary: So, I wonder if a similar thing happens if you have division inside the argument of a log function. I'm going to try some examples. If my theory works, then all of these graphs will just be vertical shifts of $y = \log_2 x$.

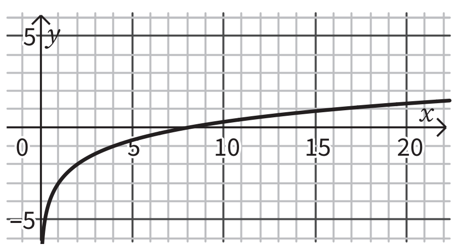
8. Here are Mary's examples and their graphs. Test Mary's theory by trying to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

   a. $y = \log_2\left(\frac{x}{4}\right)$



   Equivalent equation:

   b. $y = \log_2\left(\frac{x}{8}\right)$



   Equivalent equation:

9. Use these examples to write a rule for division inside the argument of a logarithm that is like the rule that Mary wrote for multiplication inside a logarithm.

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10. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counterexample.

Abe: You're definitely brilliant for thinking of that multiplication rule. But I'm a genius because I've used your multiplication rule to come up with a power rule. Let's say you start with:

$\log_2(x^3)$

Really that's the same as having: $\log_2(x \cdot x \cdot x)$

So, I could use your multiplying rule and write: $\log_2 x + \log_2 x + \log_2 x$

I notice there are 3 terms that are all the same. That makes it: $3\log_2 x$

So my rule is: $\log_2\left(x^3\right) = 3\log_2 x$

If your rule is true, then I have proven my power rule.

Mary: I don't think it's really a power rule unless it works for any power. You only showed how it might work for 3.

Abe: Oh, good grief! Ok, I'm going to say it can be any number, x, raised to any power, k. My power rule is:

$$\log_2(x^k) = k\log_2 x$$

Are you satisfied?

11. Provide an argument about Abe's power rule. Is it true or not?

Abe: Before we win the Nobel Prize for mathematics, I suppose that we need to think about whether or not these rules work for any base.

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12. The three rules, written for any base $b > 1$ are:

Logarithm of a Product Rule: $\log_b xy = \log_b x + \log_b y$

Logarithm of a Quotient Rule: $\log_b \frac{x}{y} = \log_b x - \log_b y$

Logarithm of a Power Rule: $\log_b x^k = k \log_b x$

Make an argument for why these rules will work in any base $b > 1$ if they work for base 2.

Ready for More?

Find an example that demonstrates the logarithm rules in bases other than base 2. For instance, an illustration of the Logarithm of a Product Rule in base 3 might be:

$\log_3 27 = 3$ and $\log_3 (3 \cdot 9) = \log_3 3 + \log_3 9 = 1 + 2 = 3$

Takeaways

Properties of Logarithms:

Lesson Summary

In this lesson, we examined graphs to find equivalent expressions for logarithmic functions. We

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justified three logarithm properties that are true for any logarithm base. They are:

Logarithm of a Product Rule: $\log_b(xy)=\log_b x+\log_b y$

Logarithm of a Quotient Rule: $\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$

Logarithm of a Power Rule: $\log_b(x^k)=k\log_b x$

Retrieval

1. Rewrite the expression using exponents. $\sqrt[3]{(54x^6)}$

2. Rewrite $\log_2 \sqrt[3]{8}$ with a fractional exponent.

3. Rewrite the equation $6^5 = 7{,}776$ in logarithmic form.

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