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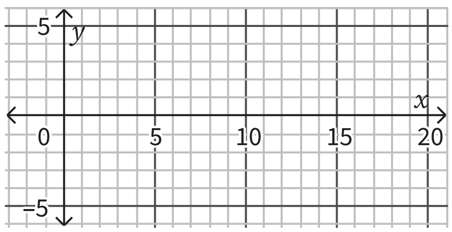
Lesson 4: Log-Arithm-etic

Practice Understanding

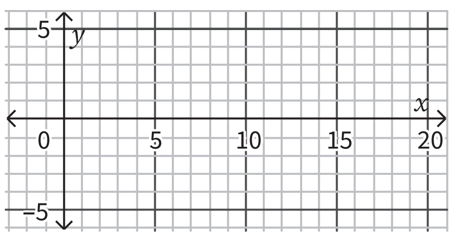
Jump Start

Graph each of the following functions:

a. $f(x) = \log_2(x+1)$



b. $g(x) = 2\log_2(x+1)$



Learning Focus

Use logarithm properties to find equivalent algebraic expressions.

Use logarithm properties to find values for logarithmic expressions using other known values.

How can the properties of logarithms help us in working with logarithmic expressions?

Open Up the Math

Launch, Explore, Discuss

Abe and Mary are feeling good about their log rules and bragging about their mathematical prowess to all their friends when this exchange occurs:

Stephen: I guess you think you're pretty smart because you figured out some log rules, but I want to know what they're good for.

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Abe: Well, we've seen a lot of times when equivalent expressions are handy. Sometimes when you write an expression with a variable in it in a different way, it means something different.

1. What are some examples from your previous experience where equivalent expressions were useful?

Mary: I was thinking about the *Log Logic* task where we were trying to estimate and order certain log values. I was wondering if we could use our log rules to take values we know and use them to find values that we don't know.

For instance: Let's say you want to calculate $\log_2 6$. If you know what $\log_2 2$ and $\log_2 3$ are, then you can use the product rule and say:

$$\log_2 (2 \cdot 3) = \log_2 2 + \log_2 3$$

Stephen: That's great. Everyone knows that $\log_2 2 = 1$, but what is $\log_2 3$?

Abe: Oh, I saw this somewhere. Uh, $\log_2 3 = 1.585$. So Mary's idea really works. You say:

$$\begin{aligned} \log_2 (2 \cdot 3) &= \log_2 2 + \log_2 3 \\ &= 1 + 1.585 \\ &= 2.585 \\ \log_2 6 &= 2.585 \end{aligned}$$

2. Based on what you know about logarithms, explain why 2.585 is a reasonable value for $\log_2 6$.

Stephen: Oh, oh! I've got one. I can figure out $\log_2 5$ like this:

$$\begin{aligned} \log_2 (2 + 3) &= \log_2 2 + \log_2 3 \\ &= 1 + 1.585 \\ &= 2.585 \\ \log_2 5 &= 2.585 \end{aligned}$$

3. Can Stephen and Mary both be correct? Explain who's right, who's wrong (if anyone) and why.

Now you can try applying the logarithm rules yourself. Use the values that are given and the ones that you know by definition, like $\log_2 2 = 1$, to figure out each of the following values. Explain what you did in the space below each problem.

$\log_2 3 = 1.585$

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$\log_2 5 = 2.322$

$\log_2 7 = 2.807$

The three rules, written for any base $b > 1$ are:

Logarithm of a Product Rule: $\log_b xy = \log_b x + \log_b y$

Logarithm of a Quotient Rule: $\log_b \frac{x}{y} = \log_b x - \log_b y$

Logarithm of a Power Rule: $\log_b x^k = k \log_b x$

4. $\log_2 9 =$

5. $\log_2 10 =$

6. $\log_2 12 =$

7. $\log_2 \left(\frac{7}{3}\right)$

8. $\log_2 \left(\frac{30}{7}\right) =$

9. $\log_2 486 =$

10. Given the work you have just done, what other values would you need to figure out the value of the base 2 logarithm for any rational number?

Sometimes thinking about equivalent expressions with logarithms can get tricky. Consider each of the following expressions and decide if they are always true for the numbers in the domain of the logarithmic function, sometimes true, or never true. Explain your answers. If you answer “sometimes true,” describe the conditions that must be in place to make the statement true.

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11. $\log_4 5 - \log_4 x = \log_4\left(\frac{5}{x}\right)$

12. $\log 3 - \log x - \log x = \log\left(\frac{3}{x^2}\right)$

13. $\log x - \log 5 = \frac{\log x}{\log 5}$

14. $5 \log x = \log x^5$

15. $2 \log x + \log 5 = \log\left(x^2 + 5\right)$

16. $\frac{1}{2} \log x = \log \sqrt{x}$

17. $\log\left(x - 5\right) = \frac{\log x}{\log 5}$

Ready for More?

You have already figured out values like $\log_2 6$ and you know some values like $\log_2 4$, what other values could you figure out?

Create some more logarithmic expressions that can be calculated. Challenge another student with your logarithmic expression and then try to figure out the logarithmic expression that they created.

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Takeaways

Strategies for working with logarithmic expressions and using properties of logarithms:

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Lesson Summary

In this lesson, we learned to find the value of a logarithmic expression combining known values and using the logarithm properties. We determined if certain logarithmic equations were true and reinforced that the logarithm product and quotient rules are only for multiplication and division inside the argument of the logarithm.

Retrieval

Factor out the greatest common factor in the expression. Then rewrite the numbers inside the parentheses.

1. $12 + 12\,(0.75)$
2. $200 - 200\,(0.4)$

3. Use your calculator and the definition of $\log x$ (recall the base is 10) to find the value of x. (Round your answers to four decimals.)

   $\log x = 2.95$