NAME DATE PERIOD

Lesson 5: Common Sense

Solidify Understanding

Learning Focus

Use logarithms to solve exponential equations.

Solve systems of equations that contain exponential functions.

How can we use logarithms and algebraic reasoning to help us solve exponential equations?

Open Up the Math

Launch, Explore, Discuss

You already know that our number system is base 10, so each of the different place values in a number are powers of 10. That makes the base 10 exponential and base 10 logarithmic functions very important. Because the base 10 logarithm is so commonly used, it is called the "common log," and to make the notation a little more compact, it is generally written without the base. So, it is used like this:

$\log 10 = 1$, because $10^1 = 10$

$\log 100 = 2$, because $10^2 = 100$

See how the base is just assumed to be 10 when a base isn't written? This is the kind of concise notation that mathematicians just love! When you use the log key on your calculator, it is automatically base 10. (Some technology allows you to enter a different base for a logarithm also.)

Each of the sections below contains puzzles for you to solve about base 10 exponential functions. You will be using base 10 logarithms and your knowledge of exponents to find missing values for exponential functions in tables, graphs, and equations. As you are working, watch for strategies that will help you to solve equations that have a variable in the exponent.

Table Puzzles

1. Find the missing values of x in the tables.

NAME DATE PERIOD

a.

x	$y = 10^x$
-2	$\frac{1}{100}$
1	10
	50
	100
3	1000

b.

x	$y = 3(10^x)$
	0.3
0	3
	94.87
2	300
	1503.56

c. When you were trying to find x when y was 50 in the first table, you were probably thinking of an equation like $50 = 10^x$, even if you didn't write it down. Here's your chance! Write all the equations and their solutions that were used to find the missing x in both tables. I gave you the first one. You're welcome!

Table **a**: $50 = 10^x$,

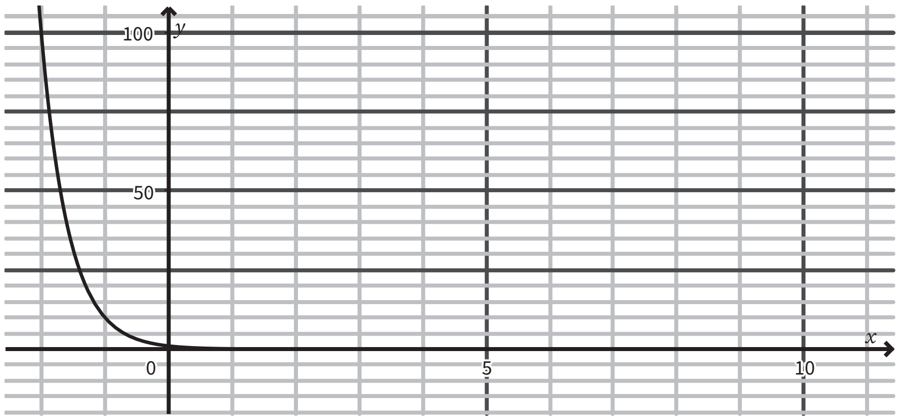
Table **b**:

2. What strategy did you use to find the solutions to these equations when you were filling in the tables?

Graph Puzzles

3. The graph of 10^{-x} is given below. Use the graph to solve the equations for x and label the solutions.

NAME DATE PERIOD



a. $10^{-x} = 40$

$x =$

Label the solution with an A on the graph.

b. $10^{-x} = 10$

$x =$

Label the solution with a B on the graph.

c. $10^{-x} = 0.1$

$x =$

Label the solution with a C on the graph.

4. Let's look a little closer at the solutions that you obtained from the graph. Consider the equation:

$10^{-x} = 10$

Would you get the same result if you took the base 10 logarithm of both sides of the equation? Try it here:

$10^{-x} = 10$

$\log_{10} 10^{-x} = \log_{10} 10$

Keep going now by rewriting both sides, using logarithm properties.

NAME DATE PERIOD

5. Let's try it again with $10^{-x} = 0.1$. Start by taking the base 10 logarithm of both sides of the equation, then write equivalent expressions and check your answer with the graph.

   $10^{-x} = 0.1$

6. Why does this process give the same value as the graph?

7. One of the equations you wrote in the table puzzles was: $94.87 = 3\left(10^x\right)$.

   How could you unwind this equation using basic operations and logarithms? Show your steps here.

Now you're ready for the equation puzzles. Here we go!

Equation Puzzles

Solve each equation for x.

8. $10^x = 10,000$

9. $125 = 10^x$

10. $10^{x+2} = 347$

11. $5\left(10^{x+2}\right) = 0.25$

12. $10^{-x-1} = \frac{1}{36}$

13. $-\left(10^{x+2}\right) = 16$

NAME DATE PERIOD

Combo Puzzles

Choose any method to solve.

14. $\begin{cases} y = 10^x \\ y = 100 \end{cases}$

15. $\begin{cases} y = 10^x - 50 \\ y = 25 \end{cases}$

16. $\begin{cases} y = 10^{-x} \\ y = x + 5 \end{cases}$

Ready for More?

Solve the equation $2^{x-1} = 30$ using two methods:

Method 1: Use a base 2 logarithm to find an exact expression for the solution.

Method 2: Use a base 10 logarithm to find an approximate value for the solution.

NAME DATE PERIOD

Takeaways

Strategies for solving exponential equations and systems:

Vocabulary

- common logarithm

Bold terms are new in this lesson.

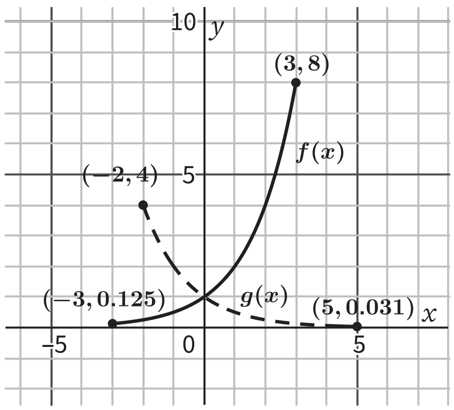
Lesson Summary

In this lesson, we found solutions to base 10 exponential equations by using base 10 logarithms to undo the equation. We used tables and graphs to help support our thinking about whether the solutions we find are reasonable. We solved systems of equations both by finding the intersection of the graphs and by finding the value that makes both equations true.

Retrieval

1. The graphs of $f\left(x\right)=2^x$ and $g\left(x\right)=2^{-x}$ (both with restricted domains) are shown in the same coordinate plane.

NAME DATE PERIOD



a. Make a list of the features of the function $f\left(x\right)$.

b. Make a list of the features of $g\left(x\right)$.

2. Use long division to calculate $\left(1{,}568 \div 13\right)$. Show each step.

Write your answer as a quotient and a remainder.