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Lesson 6: Compounding the Problem

Develop Understanding

Learning Focus

Understand how different factors affect the amount earned from compound interest.

Determine the features of a new function: $f(x) = e^x$.

Which factor makes the most difference in the amount of money earned in a savings account: the interest rate, the number of compounding periods per year, or the number of years invested?

Open Up the Math

Launch, Explore, Discuss

Part I: As an enterprising young mathematician, you know that your superior knowledge of mathematics will help you make better decisions about all kinds of things in your life. One important area is money $$$. So, you've been contemplating the world and wondering how you could maximize the money that you make in your savings account.

You're young and you haven't saved much money yet. As a matter of fact, you only have $100, but you really want to make the best of it. You like the idea of compound interest, meaning that the bank pays you interest on all the money in your savings account, including whatever interest that they had previously paid you. This sounds like a very good deal. You even remember that the formula for compound interest is exponential. Let's see, it is:

$$A = P(1 + \frac{r}{n})^{nt}$$

Where A = the amount of money in the account at any time t

P = the principal, or the original amount invested in the account

r = the annual interest rate

n = the number of compounding periods each year

t = the number of years

1. If your saving account pays a generous 5% per year and is compounded only once each year, how much money would be in the account at the end of 1 year?

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2. How much money would be in the account at the end of 20 years?

It seems like the more compounding periods in the year, the more money that you should make. The question is, does it make a big difference?

3. Compare the amount of money that you would have after 20 years if it is compounded 2 times each year (semi-annually), 4 times per year (quarterly), 12 times per year (monthly), 365 times per year (daily), and then hourly. Find a way to organize, display, and explain your results to your class.

It turns out that the value you found in your compounding problem is 100 times a very famous irrational number, named e. Because e is irrational it is a nonterminal, nonrepeating decimal number, like π. The first few digits of e are $2.7182818284590452353602874713527$. Like π, e is a number that occurs in the mathematics of many real-world situations, including exponential growth. One of the formulas using e results from the thinking that you just did about compound interest. It can be shown that the amount of money A in a savings account where money is compounded continuously is given by:

$$A = Pe^{rt}$$

P = the principal, or the original amount invested in the account

r = the annual interest rate

t = the number of years

4. It is fairly typical for savings accounts to be compounded monthly. Compare the amount of money in two savings accounts after 10 years with the same initial investment of $\$500$ and interest rate of 3% in each account if the first account is compounded monthly and the second account is compounded continuously.

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5. Use technology to compare the graphs of the two accounts. What conclusions would you draw about the effect of changing the number of compounding periods on a savings account?

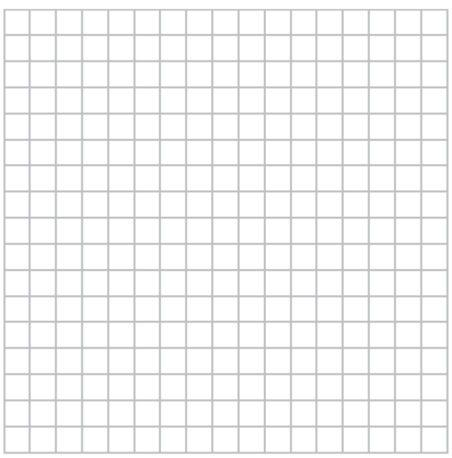
Part II

Since e is widely used to model exponential growth and decay in many contexts, let's get a little more familiar with the base e exponential function:

$$f\left(x\right)=e^{x}$$

6. Make a prediction about the graph of $f(x)$. Explain what prior knowledge you used to make your prediction.

7. Create a table and a graph and describe the mathematical features of $f(x)$.



Ready for More?

If you could invest your $\$100$ at 5% compounded quarterly, which change would have the biggest effect on the amount of money earned after 20 years?

a. Changing the number of compounding periods from quarterly to daily?

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b. Changing the interest rate from 5% to 5.5%?

c. Changing the amount invested to $\$150$?

Takeaways

Features of $f(x) = e^x$:

Adding Notation, Vocabulary, and Conventions

$e \approx$

e is

Vocabulary

- continuous compound interest
- explicit equation
- irrational number
- recursive equation

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we explored the effects of changing the number of compounding periods in an investment that earns compound interest. In the course of this exploration, we found the number e, an irrational number that arises in many exponential growth and decay contexts. We investigated the function $f(x) = e^x$ and identified the features, finding that the behavior of the function is very similar to $y = 2^x$, a function with which we have a lot of experience.

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Retrieval

1. Use first and second differences to identify the type of function represented by the pattern in each table. If the pattern in the table is linear, write both the explicit and the recursive equations. If the pattern is quadratic, write only the recursive equation.

   a. **Table 1:**

n	$f(n)$
0	4
1	6
2	8
3	10
4	12

   b. **Table 2:**

n	$g(n)$
0	2
1	8
2	16
3	26
4	38

2. Fill in the blanks using what you know about logarithms.

   I know that $\log_7 343 = 3$ and $\log_9 81 = 2$ because $343 =$ ____________ and $81 =$ ____________. If the argument of the logarithm is the base raised to a power, then the answer to $\log_b b^a$ is ________________________.