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Lesson 7: Logs Go Viral

Practice Understanding

Learning Focus

Model continuous growth and decay using base e exponential functions.

Solve exponential equations using natural logarithms.

How can we solve continuous growth and decay problems that are modeled using base e?

Open Up the Math

Launch, Explore, Discuss

As we learned in the previous lesson, *Compounding the Problem*, when money is compounded continuously, it turns out that the base of the exponential growth function is $e \approx 2.71828$. It is true that basically all situations that grow or decay continuously can be modeled with a base e exponential function. The basic formulas are:

Continuous growth: $A = Pe^{rt}$

Continuous decay: $A = Pe^{-rt}$

In both cases, A is the amount of "stuff" at time t. P is the amount of "stuff" to start with, or at $t = 0$, and r is the rate of growth or decay. So, if we're talking about money, A and P are dollar amounts. If we're talking about a population, A and P are the number of people in the population. If we're talking about radioactive decay, then A and P are the mass of the radioactive substance. In other words, P is what we started with and A is what we have after it grows or decays.

Since there are many things that grow or decay continuously in nature, e is called the natural exponential function and the base e logarithm is called the natural logarithm. Think about the growth of bacteria. One cell splits into two cells. The two cells begin to split, and the resulting cells split and so on. In a group of many cells, there would be a cell splitting nearly continuously, thus it is a base e exponential function. So, to use this base e exponential and its inverse, imagine the following scenarios:

You are an epidemiologist, a person who studies the outbreak and spread of diseases. Part of your job is to help avoid a pandemic—a worldwide outbreak of a disease. You know some of the most difficult diseases to deal with are viruses because they don't respond to many of the medicines we have available and because viruses are able to mutate and change quickly, making it more difficult to contain them. You have been studying a new virus that causes people to break out in spots. Suddenly, a colleague rushes into your office to inform you there is a confirmed outbreak of the virus in Europe. The growth of the virus through a population is continuous (until it is somehow

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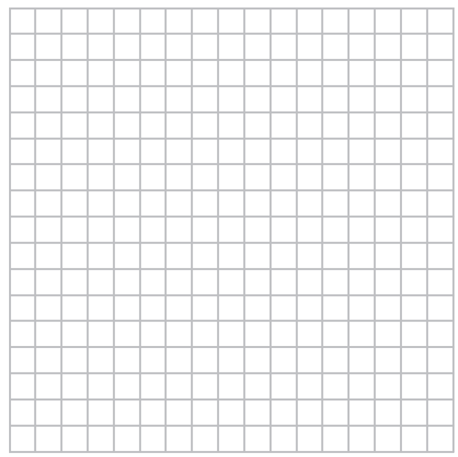
contained) at a rate of 3% per day. The current outbreak has 5 confirmed victims.

1. How many people will be infected with the virus on day 30?

2. Create a model of the spread of the spotted virus in this region if it is not contained. To simplify your model slightly, consider the 5 victims as the number of victims on day 0.

   Table:

   Equation:



3. When does your model predict there will be 50 victims? Show how you arrived at your answer.

4. Will the number of days it will take the virus to claim 100 victims be double the number of days that it took to claim 50 victims? Why or why not?

5. Calculate the number of days it will take for the virus to claim 100 victims.

6. On what day will there be 150 victims?

Now you have received a report of a mysterious illness that seems to turn the infected humans into

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mindless zombies has broken out in a major American city. Since the hungry zombies prey upon innocent people, the outbreak grows continuously at a rate of 12% per day. The outbreak begins with 80 people.

7. How many zombies will there be after 5 days?

8. How many days will it take for the zombie population to reach $3{,}700{,}000$ (about the population of Los Angeles, CA)?

9. At what rate would the zombie population be growing if it reached $190{,}000 \text{ people}$ (about the population of Salt Lake City, Utah) in 20 days?

Now we're going to get a little more far-fetched in the scenario. Let's say that zombies produce radioactive goo that decays continuously with a half-life of 3 years. (That's one more danger of having zombies around.) The half-life tells us that after 3 years, only half of the amount of goo we started with is remaining.

10. If we start with 10 pounds of zombie goo, how much will be remaining after 5 years?

11. How long will it take for the amount of zombie goo to decay to an amount less than 0.5 pounds?

12. When will there be no zombie goo left?

Ready for More?

Graph $y = \ln x$.

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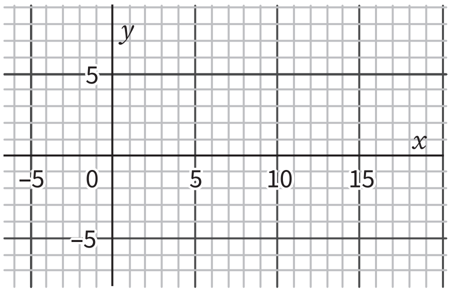

Takeaways

Solving an exponential equation with a variable in the exponent:

Adding Notation, Vocabulary, and Conventions

The inverse of is $y = e^x$ is

$\ln x$ is

Natural logarithms are used to

Vocabulary

- **natural logarithm**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we modeled continuous growth and decay using the formula that is a base e exponential function. We learned to use the base e logarithm, called the natural log, to solve exponential equations when the variable is in the exponent. The natural logarithm is written, $\ln x$,

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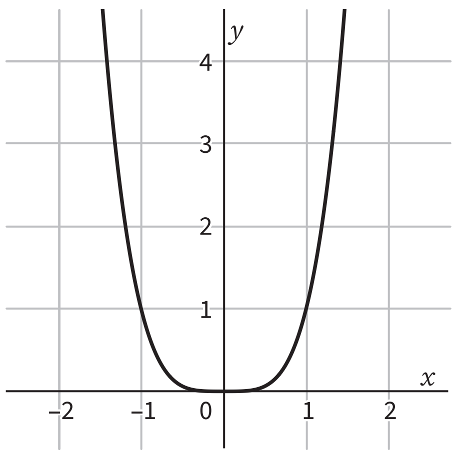
and has a dedicated function on most calculators.

Retrieval

1. How do you know that the graph of $y = x^4$ it is NOT the graph of $y = x^2$?



2. Use the properties of logarithms and the given values to find the value of the indicated logarithm.

   Given: $\log 3 \approx 0.377$ and $\log 5 \approx 0.699$

   Find: $\log \frac{9}{125}$